

## Observation prospects of leptonic and Dalitz decays of pseudoscalar quarkonia

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# Observation prospects of leptonic and Dalitz decays of pseudoscalar quarkonia

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ABSTRACT: Two types of pseudoscalar quarkonium electromagnetic decay processes, i.e. decay to a lepton pair, and to a lepton pair plus a photon (Dalitz decay), are analyzed at the leading order in NRQCD expansion. The former type of processes, highly suppressed in the Standard Model, have been hoped to act as the sensitive probes of the possible new physics. The latter type of processes generally possess much greater decay rates than the former, owing to several conspiring factors. The recently launched BES-III program, with  $10^8$   $\eta_c$  samples to be anticipated in the coming years, may be able to observe the Dalitz decays  $\eta_c \rightarrow e^+e^-\gamma$  and  $\eta_c \rightarrow \mu^+\mu^-\gamma$ , which have branching ratios of order  $10^{-6}$ . When the radiated photon becomes very soft, the Dalitz decay events will be experimentally tagged as the exclusive lepton pair events. It is found that, those quasi-two-body events that arise from  $\eta_c \rightarrow e^+e^-\gamma$  with photon energy less than the minimum sensitivity of the electromagnetic calorimeter, can vastly outnumber the literal  $\eta_c \rightarrow e^+e^-$  events, however this amplification is still not dramatic enough for the BES-III experiment to establish these events. Consequently, the expectation of looking for new physics signature in the  $\eta_c \rightarrow l^+l^-$  channel is obscured, unless the contamination from  $\eta_c \rightarrow l^+l^-\gamma$  has been taken into account carefully.

KEYWORDS: Asymptotic Freedom, Heavy Quark Physics, Electromagnetic Processes and Properties, QCD

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## 1 Introduction

Charge-neutral pseudoscalar meson decay to a lepton pair,  $P \rightarrow l^+l^-$ , has long been an interesting topic. In particular, for  $P$  to be a light pseudoscalar meson ( $\pi^0$ ,  $\eta$ ,  $\eta' \dots$ ), extensive theoretical and experimental efforts have been conducted since Drell initiated the study of  $\pi^0 \rightarrow e^+e^-$  in 1959 [1–9]. Studies of these decays can offer insights into the nonperturbative structure of the pseudoscalar mesons, in particular, help one to glean more knowledge about the  $P\gamma^*\gamma^*$  transition form factor. This type of electromagnetic decay processes are suppressed by two additional powers of  $\alpha$  with respect to  $P \rightarrow \gamma\gamma$ , and also penalized by helicity conservation. As a consequence, the decay probabilities are generally very tiny within the Standard Model (SM), rendering experimental detection rather challenging. On the other hand, the rareness of these decay processes may turn into a virtue, that is, they might be utilized as the sensitive probes of possible new interactions beyond SM.

Another intimately related type of electromagnetic decays are  $P \rightarrow l^+l^-\gamma$ . These kinds of pseudoscalar meson decay processes are of use to extract the information about the  $P\gamma^*\gamma$  form factor. Despite radiating off an extra photon, these bremsstrahlung leptonic decays in general occur much more copiously than  $P \rightarrow l^+l^-$ . For example, the so-called Dalitz decay process [10],  $\pi^0 \rightarrow e^+e^-\gamma$ , which has been observed decades ago, has a branching ratio of  $(1.198 \pm 0.032)\%$  [11]. This is more than five orders of magnitude greater than the branching fraction of  $\pi^0 \rightarrow e^+e^-$ ,  $(7.48 \pm 0.38) \times 10^{-8}$ , which was recently measured in the

KTeV E799-II experiment at Fermilab [12]. This striking disparity can be attributed to a number of facts, that such bremsstrahlung leptonic decay processes are suppressed with respect to  $P \rightarrow \gamma\gamma$  by only one additional power of  $\alpha$ , suffer no helicity suppression, and also enjoy collinear enhancement brought in by photon fragmentation to a lepton pair.

In this work, we aim to investigate both types of leptonic decay processes for  $P$  to be a pseudoscalar heavy quarkonium state, i.e.,  $\eta_Q \rightarrow l^+l^-$  and  $\eta_Q \rightarrow l^+l^-\gamma$  ( $Q = c, b$ ).<sup>1</sup> One strong incentive stems from the experimental side. For instance, the recently launched BES-III experiment, plans to accumulate an unprecedentedly large data set of charmonia, e.g.,  $10^{10} J/\psi$  and  $3 \times 10^9 \psi(2S)$  in the coming years [13]. An enormous number of  $\eta_c$  and  $\eta_c(2S)$  are expected to be produced via the radiative transitions from these  $J/\psi$  or  $\psi(2S)$  samples. Analogously, the scheduled Super BELLE experiment, will also be capable of collecting a tremendous number of  $\eta_b$  samples. Further, an even larger data set of pseudoscalar quarkonia are expected to be produced at the CERN Large Hadron Collider (LHC), though the copious backgrounds in hadron machine renders the detection of such rare decays rather challenging. In any event, it seems, at the current time, not of only academic interests to assess the observation potentials of these rare electromagnetic decay processes in the forthcoming experiments.

From a theoretical perspective, it is also worthwhile to study these rare electromagnetic decays of pseudoscalar quarkonia, to enrich our knowledge about heavy quark physics. A heavy quarkonium state, being a heavy-quark heavy-antiquark pair tightly bound via the strong interaction, is the best understood among all types of hadrons. In sharp contrast to light meson decay, which must be analyzed by some nonperturbative tools, quarkonium decay can be accommodated in the perturbative QCD framework, owing to the condition  $m_Q \gg \Lambda_{QCD}$ . Indeed, annihilation decays of heavy quarkonium, especially the electromagnetic ones that we plan to investigate, can be systematically tackled by the modern effective-field-theory formalism, the nonrelativistic QCD (NRQCD) factorization approach [14]. In a quarkonium electromagnetic decay process, this factorization approach allows one to systematically separate the hard quantum fluctuation of order heavy quark mass  $m$  from the low-energy contributions of order  $mv$  or smaller, where  $v$  signifies the typical velocity of  $Q$  or  $\bar{Q}$  in a quarkonium. Empirically,  $v^2 \approx 0.3$  for charmonium, and 0.1 for bottomonium.

We note that, both types of leptonic decay processes of  $\eta_Q$  have already been partly investigated by different authors. For instance,  $\eta_Q \rightarrow l^+l^-$  has been studied in ref. [4, 15, 16], and the Dalitz decay  $\eta_Q \rightarrow l^+l^-\gamma$  was considered in [17, 18]. Nevertheless, a comprehensive analysis based on the NRQCD factorization approach is still lacking.<sup>2</sup> For this reason, we feel that it might be rewarding to revisit these two processes from this angle.

<sup>1</sup>In literature, the term *Dalitz decay* has been specifically reserved for  $\pi^0 \rightarrow e^+e^-\gamma$  [10]. In this work we generalize the use of this term, i.e. we also use it to refer to any of the  $\eta_Q \rightarrow l^+l^-\gamma$  process.

<sup>2</sup>Ref. [4] studies the  $\eta_Q \rightarrow l^+l^-$  process within a bound state quark model, which nevertheless may be viewed as a primitive version of the NRQCD approach. Ref. [15] employs the NRQCD factorization explicitly, but only studies a few processes using numerical recipe. Both of works completely neglect the weak interaction contribution, which turns out to be inadequate for  $\eta_b$  decay.

We will work at the leading order in NRQCD expansion only. However, the systematics of this approach renders future implementation of higher-order corrections possible.

We summarize the main outcome of this work. We confirm the analytic expression for the electromagnetic contribution to the decay  $\eta_Q \rightarrow l^+l^-$ , which was first reported in ref. [4]. We also include the often-omitted weak interaction contribution arising from  $Z^0$  exchange. It is found that, although the weak interaction plays a negligible role in the leptonic decay of  $\eta_c$ , its effect can become important in  $\eta_b$  decay, especially for  $\eta_b \rightarrow \tau^+\tau^-$ .<sup>3</sup> Despite this, the net SM predictions to the decay rates are still too suppressed for these processes to be observed experimentally in the foreseeable future.

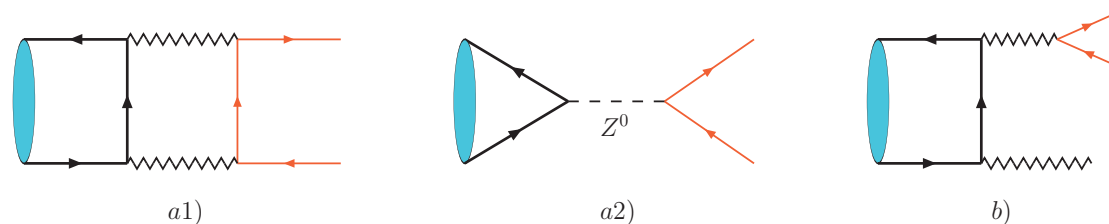
We also perform a comprehensive study of numerous pseudoscalar quarkonium Dalitz decay processes, and verify that such decays generally possess much more enhanced decay probability than  $\eta_Q \rightarrow l^+l^-$ . In particular, it is found that  $\eta_c \rightarrow e^+e^-\gamma$ ,  $\mu^+\mu^-\gamma$ , with branching fractions of order  $10^{-6}$ , might have bright prospect to be observed at the BES-III experiment. We also analyze the energy distributions of lepton and photon in these Dalitz decay processes. We hope future measurements of these energy spectra can test our predictions critically.

We emphasize that any realistic electromagnetic calorimeter is limited by the finite sensitivity to detect soft photons. This indicates that, the Dalitz decay event  $\eta_Q \rightarrow l^+l^-\gamma$  will fake the literal  $\eta_Q \rightarrow l^+l^-$  event, if the emitted photon is too soft to be registered by the electromagnetic calorimeter. Therefore, the hope of seeking new interaction beyond SM in the  $\eta_Q \rightarrow l^+l^-$  channel becomes obscured, unless the contamination from the respective Dalitz decay is thoroughly understood and incorporated in the analysis. Our study shows that, the number of quasi-two-body events that arise from  $\eta_c \rightarrow e^+e^-\gamma$  with photon energy restricted to be less than 20 MeV, can easily surpass that from  $\eta_c \rightarrow e^+e^-$  by two orders of magnitude, but such enhancement is still not significant enough to warrant the establishment of such events at BESIII experiment.

The remainder of the paper is distributed as follows. In section 2, we calculate the pseudoscalar quarkonia decays to a lepton pair at the leading order in NRQCD expansion, including both contributions from QED and weak interaction. We also compare our results with the previous ones. In section 3, we present a detailed calculation of the pseudoscalar quarkonium Dalitz decay processes at the leading order in NRQCD expansion. The inclusive energy spectra of the photon and the lepton are presented, and the QED fragmentation function for a photon to split into a lepton can be extracted. We also obtain a succinct expression for the decay branching fraction integrated over the full three-body phase space. In section 4, a comprehensive numerical analysis for numerous processes of pseudoscalar quarkonia decays to a lepton pair and the Dalitz decays are made. Particular attention is paid to the interference pattern between QED and the  $Z^0$ -exchange contributions to the former type of processes. We assess the observation prospects of both types of decay processes. We also investigate to which extent the Dalitz decay events will fake the exclusive lepton pair events. In section 5, we summarize and present an outlook.

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<sup>3</sup>The weak interaction effect for  $\eta_c \rightarrow l^+l^-$  has also been considered in [16].



**Figure 1.** Lowest-order diagrams for the processes  $\eta_Q \rightarrow l^+l^-$  [a] and  $\eta_Q \rightarrow l^+l^-\gamma$  [b]. In the former process, we represent the QED contribution by a1), the weak interaction contribution by a2). The crossed diagrams for a1) and b) have been suppressed.

## 2 Pseudoscalar quarkonium decay to a lepton pair

The purpose of this section is to derive the analytic expressions for the amplitude of  $\eta_Q \rightarrow l^-l^+$  in the lowest-order NRQCD expansion, by taking only the SM interactions into consideration. That is, we will consider the contributions from both electromagnetic and weak interactions, and investigate their interference pattern.

### 2.1 The electromagnetic contribution to $\eta_Q \rightarrow l^-l^+$

In this subsection we focus on the QED contribution to this process. Unlike the leptonic decay of vector quarkonium such as  $J/\psi$ , a pseudoscalar quarkonium cannot directly decay to a lepton pair through annihilation into a virtual photon at tree level. At the lowest order in electromagnetic and strong couplings, this process proceeds through the one-loop QED box diagram, as depicted in figure 1a).<sup>4</sup>

The validity of the NRQCD approach rests upon one of the key characteristics of quarkonium, that both of its constituents move non-relativistically in the quarkonium rest frame, so the quark relative velocity can serve as a small expansion parameter of the theory. For this reason, NRQCD would be a very poor framework to describe light mesons such as  $\pi$ ,  $\eta$ ,  $\eta'$ , etc. Nowadays the NRQCD approach has been accepted as the standard tool to analyze heavy quarkonium decay and production processes. Since hard reactions involving quarkonium necessarily probe the scale of order heavy quark mass, by appealing to the asymptotic freedom of QCD, NRQCD approach allows one to put the amplitude in a factorized form, i.e., the sum of the products of perturbatively calculable short-distance coefficients and nonperturbative but universal NRQCD matrix elements.

The calculation of quarkonium decay at the LO in NRQCD expansion is standard. We assume the quarkonium state composed of a heavy quark  $Q$  and its antiquark  $\bar{Q}$ , and abbreviate the  $Q\bar{Q}(^1S_0^{(1)})$  state by  $\eta_Q$ . We assign the momenta carried by  $\eta_Q$ ,  $l^-$ ,  $l^+$  as  $P$ ,  $p_1$ ,  $p_2$ . At the LO in velocity expansion, one can routinely obtain the amplitude for  $\eta_Q \rightarrow l^-l^+$  by first computing the amplitude for  $Q(\frac{P}{2})\bar{Q}(\frac{P}{2}) \rightarrow l^-(p_1)l^+(p_2)$ , enforcing  $Q$  and  $\bar{Q}$  to carry equal momentum, then projecting it onto the intended  $^1S_0^{(1)}$  state.

<sup>4</sup>The same annihilation type of diagram, with the lepton replaced by the light quark and the photon replaced by gluon, has been hypothesized to account for the  $\eta_c - \eta(\eta')$  mixing [19].

There are totally two lowest-order QED diagrams for this process, one of which is shown in figure 1-a1). The other undrawn diagram can be obtained from it by reversing the fermionic arrow in either the quark or the lepton line. By  $C$ -invariance, both of diagrams yield the identical results. After some straightforward calculation, we can express the electromagnetic decay amplitude at the LO in NRQCD expansion as

$$\mathcal{M}_{\text{EM}}[\eta_Q \rightarrow l^- l^+] = 2\sqrt{2N_c} e_Q^2 e_l^2 \alpha^2 \frac{\psi_{\eta_Q}(0)}{m_Q^{5/2}} f\left(\frac{m_l^2}{m_Q^2}\right) [m_l \bar{u}(p_1) \gamma_5 v(p_2)], \quad (2.1)$$

where  $N_c = 3$  is the number of colors.  $e_l$  and  $e_Q$  signify the electric charges of charged lepton and heavy quark in units of  $|e|$  ( $e_l = -1$ ,  $e_c = 2/3$  and  $e_b = -1/3$ ), and  $\alpha$  is the fine structure constant. The nonperturbative factor  $\psi_{\eta_Q}(0)$ , is the wave function at the origin for the  $\eta_Q$  state, which can be identified with the LO NRQCD matrix element.  $m_l$  and  $m_Q$  denote the masses of lepton and quark, respectively. At the current level of accuracy, it is legitimate to treat  $m_Q$  and  $M_{\eta_Q}/2$  interchangeably. The dimensionless function  $f$  encodes the effect of loop contribution, and is normalized in such a way that it depends on the ratio of lepton mass to quark mass at most logarithmically.

Some remarks on the traits of this process are in order. Because the lepton pair must form a  $^1S_0$  state to conserve angular momentum, a  $\gamma_5$  is expected to be sandwiched between the leptonic spinors in the decay amplitude. Indeed, with the aid of Dirac equation, one easily verifies that the bispinor  $\bar{u}(p_1) \gamma_5 v(p_2)$  exhausts all the possible Lorentz structures. It is well known that leptonic decays of pseudoscalar meson suffer from the so-called *helicity suppression*, and would be strictly forbidden when lepton mass set to zero, hence there should be an explicit factor of lepton mass appearing in the amplitude.<sup>5</sup> Note that the  $f$  function diverges with  $m_l$  only logarithmically, so equation (2.1) is compatible with the helicity suppression mechanism.

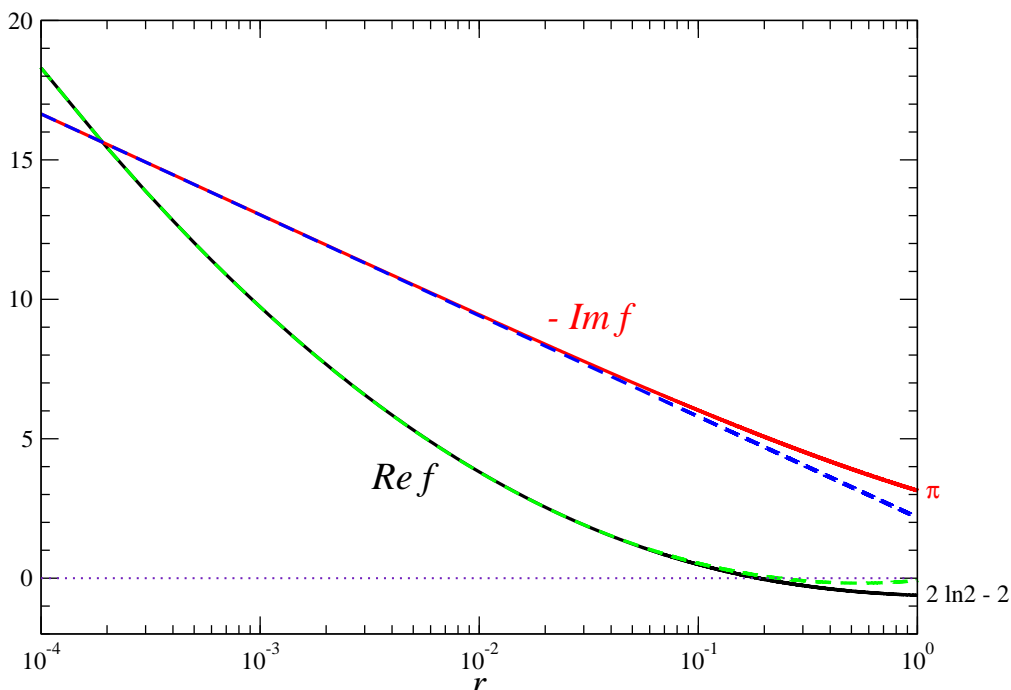
It turns out that the evaluation of the box diagram can be reduced to evaluating a three-point one-loop integral. The encountered loop integrals are both ultraviolet and infrared finite, hence may be directly computed at four spacetime dimension. After some efforts, we can obtain the closed form for the  $f$  function:

$$f(r) = \frac{1}{\beta} \left[ \frac{1}{4} \ln^2 \left( \frac{1+\beta}{1-\beta} \right) - \ln \left( \frac{1+\beta}{1-\beta} \right) + \frac{\pi^2}{12} + \text{Li}_2 \left( -\frac{1-\beta}{1+\beta} \right) - \frac{i\pi}{2} \ln \left( \frac{1+\beta}{1-\beta} \right) \right], \quad (2.2)$$

where  $\text{Li}_2$  is the dilogarithm. We have introduced  $r \equiv \frac{4m_l^2}{M_{\eta_Q}^2} = \frac{m_l^2}{m_Q^2}$ , and  $\beta \equiv \sqrt{1-r}$  is the velocity of the outgoing lepton in the  $\eta_Q$  rest frame. The shape of this function is shown in figure 2. Note eq. (2.2) is identical to the  $R$  function given in equation (12) of [4], once the Spence function  $\Phi(x)$  there is identified with  $-\text{Li}_2(-x)$ . It may be worth pointing out that, in order to reach the compact expression given in (2.2), we have made use of the

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<sup>5</sup>We stress this phenomenon not only pertains to pseudoscalar meson leptonic decay. The decay  $\chi_{c0} \rightarrow e^+ e^-$  should also be forbidden in the zero- $m_e$  limit, again due to the conflict between angular momentum conservation and helicity conservation in massless QED.



**Figure 2.** The profile of the function  $f(r)$ . Solid lines represent the exact results given in (2.2), and the dashed ones represent the asymptotic expressions as given in (2.3a) and (2.3b).

following somewhat inapparent relation:<sup>6</sup>

$$\text{Li}_2\left(\frac{1-\beta}{2}\right) + \text{Li}_2\left(-\frac{1-\beta}{1+\beta}\right) + \frac{1}{2} \ln^2\left(\frac{1+\beta}{2}\right) = 0.$$

The occurrence of the imaginary part in (2.2) is linked with the “unitarity bound” of the branching ratio, which is obtained from tying the amplitudes of  $\eta_Q \rightarrow \gamma\gamma$  and  $\gamma\gamma \rightarrow l^-l^+$  together according to the cutting rule.

In Nature heavy quarks are generally much heavier than leptons, especially for the first two generations. It is then useful to know the asymptotic behavior of the  $f$  function in the  $r \rightarrow 0$  limit:

$$\text{Re}f_{\text{asym}}(r) = \frac{1}{4} \ln^2 r + (1 - \ln 2) \ln r + \ln^2 2 - 2 \ln 2 + \frac{\pi^2}{12} + \mathcal{O}(r \ln^2 r), \quad (2.3a)$$

$$\text{Im}f_{\text{asym}}(r) = \pi \left( \frac{1}{2} \ln r - \ln 2 + \mathcal{O}(r \ln r) \right). \quad (2.3b)$$

<sup>6</sup>This relation can be proven with the help of the identity  $\text{Li}_2\left(\frac{x}{2}\right) = \text{Li}_2(x) + \text{Li}_2\left(\frac{1}{2}\right) + \text{Li}_2\left(\frac{x}{2(x-1)}\right) + \text{Li}_2(x-1) + \frac{1}{2} \ln^2(2-2x)$ . Substituting  $x = 1 - \beta$  and  $x = -\frac{1-\beta}{1+\beta}$  into this identity separately, taking the respective difference, one then obtains the desired answer.



As anticipated,  $f(r)$  depends on  $r$  only logarithmically. From figure 2, one can see that the function  $f_{\text{asym}}(r)$  already constitutes a rather good approximation as  $r \leq 0.1$ . To reproduce these asymptotic behaviors more efficiently, one may appeal to the *method of region* [20] by dismembering the original loop integration into the sum of integrations from different regions, e.g., hard, soft, collinear-to- $l^-$  and collinear-to- $l^+$  in our case [21]. One may identify the double logarithm in (2.3a) that originates from the overlap between collinear and soft singularities.

In passing we remark on one peculiarity reported in a recent calculation of the same process that employing the light-cone approach [16]. In that approach, the amplitude is expressed as the convolution of a hard-scattering part with the light-cone distribution amplitude of  $\eta_Q$ . The hard part there is found to scale as  $\ln r/\sqrt{r}$  in the limit  $r \rightarrow 0$ . This infrared sensitivity is theoretically disastrous because it diametrically conflicts with the requirement of helicity suppression— that this process should be strictly forbidden for a massless lepton. This nuisance, if confirmed to persist, may indicate that, the light-cone operator product expansion (OPE), which underlies the calculational framework in [16], may no longer be suited to describe heavy quarkonium decay. The light-cone expansion is usually formulated as an expansion in terms of a small energy scale like quark mass over a large momentum transfer. It should be appropriate for a high-energy exclusive heavy quarkonium production processes, as the large momentum transfer scale can be identified with the center-of-mass energy of the reaction, which may indeed be much greater than the heavy quark mass. However in the process at hand,  $m_Q$  itself already acts as the highest energy scale, so it is difficult to imagine the actual meaning of the light-cone expansion here. In a space-time picture, the slowly-moving heavy quark and antiquark typically experience an instantaneous strong force, so they are typically separated by a distance of order  $1/m_Q v$  but local in time, which is quite far from a light-like separation. In our opinion, NRQCD factorization approach, which is closely related to a local OPE by treating  $1/m_Q$  as an expansion parameter, provides the most natural and economic framework to account for the heavy quarkonium decay, in particular for the processes considered in this work.

It is also interesting to look at the alternative limit  $r \rightarrow 1$  ( $\beta \rightarrow 0$ ), where the mass of  $\eta_Q$  is just sitting at the threshold of twice lepton mass. From (2.2), one finds the following limiting value for  $f$ :

$$f(1) = -2 + 2 \ln 2 - i\pi, \quad (2.4)$$

which can also be seen in figure 2. As noted in ref. [4], this is a well-known result, which is responsible to the two-photon annihilation contribution to the hyperfine splitting between the orthopositronium and parapositronium [22] (see also [23, 24]).

## 2.2 The weak-interaction contribution to $\eta_Q \rightarrow l^- l^+$

Neutral quarkonium decay is normally not a good place to look for the trace of weak interaction, which is generally overshadowed by the strong and electromagnetic interactions. However, for the processes at hand, the LO QED amplitude has to proceed at one loop order, but the weak interaction contribution instead can start at tree level, so it is not inconceivable that the weak interaction may play some role for some of these processes.

There is only one  $s$ -channel diagram that contributes to  $\eta_Q \rightarrow l^+l^-$  from  $Z^0$  exchange, as depicted in figure 1 a2). Only the axial vector coupling of  $Z^0 f \bar{f}$  contributes to this process. At the lowest order in the velocity expansion, the weak amplitude can be expressed as

$$\mathcal{M}_{\text{WEAK}} = 2\sqrt{2N_c} \frac{\psi_{\eta_Q}(0)}{\sqrt{m_Q}} \frac{\pi\alpha g_A^l g_A^Q}{M_Z^2 \sin^2 \theta_W \cos^2 \theta_W} [m_l \bar{u}(p_1) \gamma_5 v(p_2)], \quad (2.5)$$

where  $\theta_W$  is the Weinberg angle, and  $g_A^l, g_A^Q$  denote the weak axial charge of charged lepton and quark, respectively. The weak axial charge of a fermion is equal to its 3rd component of weak isospin ( $g_A^l = -\frac{1}{2}$  for  $l = e, \mu, \tau$ ;  $g_A^c = \frac{1}{2}, g_A^b = -\frac{1}{2}$ ). We have neglected the  $\eta_Q$  mass as well as the width of  $Z^0$  in the  $Z^0$  propagator, since they are much smaller than  $Z^0$  mass for  $Q = c, b$ . Therefore the  $Z^0$  exchange can be effectively mimicked by a four-fermion contact interaction. Note there is an explicit factor of  $m_l$  in eq. (2.5), again due to helicity suppression, very similar to what occurs to  $\pi^+ \rightarrow l^+ \nu_l$ .

Comparing (2.5) with (2.1), one clearly sees that, though the weak interaction amplitude is suppressed by a factor of  $m_Q^2/M_Z^2$  relative to the electromagnetic one, it suffers less suppression by one power of  $\alpha$ . For  $\eta_b$  decay to a lepton pair, these two competing effects may become comparable in magnitude. As a consequence, in order to make reliable predictions, the weak interaction effect becomes indispensable and must be included.

### 2.3 The Standard Model prediction to $\eta_Q \rightarrow l^-l^+$

Substituting eq. (2.1) and eq. (2.5) into

$$\Gamma^{\text{SM}}[\eta_Q \rightarrow l^-l^+] = \frac{\beta}{16\pi M_{\eta_Q}} |\mathcal{M}_{\text{EM}} + \mathcal{M}_{\text{WEAK}}|^2, \quad (2.6)$$

we then get the desired partial width expected in the frame of SM.

It is convenient to introduce the *normalized* decay rate of  $\eta_Q \rightarrow l^-l^+$ , defined as eq. (2.6) normalized to the partial width of  $\eta_Q \rightarrow \gamma\gamma$ :

$$\begin{aligned} R^{\text{SM}}[\eta_Q \rightarrow l^+l^-] &\equiv \frac{\Gamma^{\text{SM}}[\eta_Q \rightarrow l^+l^-]}{\Gamma_0} \\ &= \frac{\alpha^2}{2\pi^2} \beta r \left| f(r) - g_A^Q \frac{\sqrt{2} G_F M_{\eta_Q}^2}{8 e_Q^2 \alpha^2} \right|^2, \end{aligned} \quad (2.7)$$

where we have substituted  $e_l = -1$ , and used the relation  $M_Z = (\frac{\pi\alpha}{\sqrt{2}G_F})^{1/2} \frac{1}{\sin\theta_W \cos\theta_W}$ , to condense the expression for the scaled weak amplitude ( $G_F$  is the Fermi coupling constant). An important feature is that the relative importance of weak interaction contribution increases with  $M_Q$ . The partial width of  $\eta_Q \rightarrow \gamma\gamma$  is given at the lowest order in  $v$  and  $\alpha_s$ :

$$\Gamma_0 \equiv \Gamma[\eta_Q \rightarrow \gamma\gamma] = \frac{4N_c \pi e_Q^4 \alpha^2}{m_Q^2} \psi_{\eta_Q}^2(0). \quad (2.8)$$

The advantage of introducing the  $R$  ratio in (2.7) is that the nonperturbative factor  $\psi_{\eta_c}(0)$  cancels out in the ratio, and some portions of QCD radiative and relativistic corrections

to both  $\eta_Q \rightarrow l^-l^+$  and  $\eta_Q \rightarrow \gamma\gamma$  may largely cancel. Therefore, for a LO calculation like this work, using the  $R$  ratio instead of the branching ratio is presumably more appropriate. In the situation where the  $\eta_Q$  di-photon decay has been experimentally measured, one can directly obtain the  $\mathcal{B}[\eta_Q \rightarrow l^+l^-]$  by multiplying  $\mathcal{B}_{\text{exp}}[\eta_Q \rightarrow \gamma\gamma]$  with the predicted  $R$  ratio.

### 3 Dalitz decays $\eta_Q \rightarrow l^+l^-\gamma$

It is an experimental fact that, for light pseudoscalar mesons, the branching fractions of the Dalitz decay processes are several orders of magnitude greater than those of the respective leptonic decays. Certainly, it is worthwhile to examine whether the same pattern also holds for pseudoscalar quarkonium decays or not.

We note that, an analogous pseudoscalar quarkonium strong decay process, i.e.  $\eta_Q \rightarrow q\bar{q}g$ , has been previously studied by several authors by retaining a nonzero mass for  $q$ . The analytic expressions for the energy distributions of the gluon, quark and the integrated decay rate, has been presented in ref. [25]. In the following, we will independently derive the corresponding energy spectra of the photon, lepton and the integrated decay rate for the  $\eta_Q$  Dalitz decay process. When inserting a proper color factor, the exact agreement is found between our eqs. (3.5), (3.6), (3.8) and eqs. (4), (5), (6) in ref. [25]. The process  $\eta_b \rightarrow c\bar{c}g$  has also been studied numerically in [15, 26, 27].

#### 3.1 Squared amplitude of quarkonium Dalitz decay

In contrast to the rare decay  $\eta_Q \rightarrow l^+l^-$ , the Dalitz decay  $\eta_Q \rightarrow l^+l^-\gamma$  can start at tree level in QED.<sup>7</sup> There are in total two diagrams, one of which is depicted in figure 1b). The momenta of  $\eta_Q$ ,  $l^-$ ,  $l^+$ ,  $\gamma$  are assigned as  $P$ ,  $p_1$ ,  $p_2$ ,  $p_3$ , respectively. This process first proceeds through  $\eta_Q \rightarrow \gamma\gamma^*$ , and the virtual photon then fragments into a lepton pair. If the lepton is much lighter than the quark, one expects that the decay rate is dominated by the kinematic configuration where the invariant mass of lepton pair is close to its minimum,  $2m_l$ . In other word, it is the nonzero lepton mass that cuts off the potential collinear singularity.

It might be worrisome that the amplitude may diverge in another kinematic configuration, i.e., where the photon becomes very soft, and the lepton and anti-lepton move nearly back-to-back with equal momentum. It is well known that in  $J/\psi$  decay to  $l^+l^-\gamma$ , infrared divergence does arise in the long wavelength limit of photon. This divergence is in turn canceled by including the virtual correction to  $J/\psi \rightarrow l^+l^-$ , guaranteed by the Bloch-Nordsieck theorem.<sup>8</sup> However, such mechanism obviously does not apply to our case to sweep the potential infrared divergence.

A little thought reveals that figure 1b) must be regular in the  $p_3 \rightarrow 0$  limit. It is most transparent to see this in the context of nonrelativistic effective theory. To describe an

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<sup>7</sup>Weak interaction contribution can also start at tree level, nonetheless is completely negligible for both  $\eta_c$  and  $\eta_b$  Dalitz decays.

<sup>8</sup>Experimentally,  $J/\psi \rightarrow l^+l^-\gamma$  events are selected by requiring the photon energy is greater than the characteristic detector sensitivity, say, 100 MeV in the Fermilab E760 experiment [28]. Those three-body decays with photon energy less than 100 MeV are tagged as the  $J/\psi \rightarrow l^+l^-$  events.

almost on-shell quark interacting with a soft photon, one is justified to use nonrelativistic QED (NRQED). After the incoming  $Q$  emits a soft photon, it has to annihilate with  $\bar{Q}$  into a virtual photon that splits into a lepton pair. Therefore, the emission of this soft photon has to flip the spin of  $Q$ , i.e., effectively induces a magnetic dipole transition, to convert the  $Q\bar{Q}$  pair from the initial  $^1S_0$  state to a  $^3S_1$  state. The effect of this soft-photon emission is accounted for by the operator  $\frac{ee_Q}{2m_Q}\psi^\dagger\boldsymbol{\sigma}\cdot\mathbf{B}^{\text{em}}\psi$ , where  $\psi$  denotes the Pauli spinor field for  $Q$  in NRQED. The occurrence of the magnetic field strength,  $\mathbf{B}^{\text{em}}$ , which brings forth a factor of  $p_3$ , will protect against the infrared singularity arising in the quark propagator, therefore the amplitude is infrared finite.<sup>9</sup>

At the lowest order in strong coupling and in  $v$ , the decay amplitude can be routinely obtained:

$$\mathcal{M}[\eta_Q \rightarrow l^-l^+\gamma] = 2\sqrt{2N_c}e^3e_Q^2e_l\frac{\psi_{\eta_Q}(0)}{\sqrt{m_Q}}\frac{\epsilon_{\mu\alpha\nu\beta}p_3^\alpha\epsilon_\gamma^{*\nu}P^\beta}{(P\cdot p_3)(p_1+p_2)^2}\bar{u}(p_1)\gamma^\mu v(p_2), \quad (3.1)$$

where  $\epsilon_\gamma$  signifies the photon polarization vector. We have taken the undrawn diagram into account, which yields identical contribution as figure 1b) owing to the  $C$ -invariance.

From (3.1) we are reassured that, the numerator of the amplitude does contain a factor of photon momentum, which is crucial to tame the infrared singularity arising from the quark propagator. It is also worth noting that, since the lepton and anti-lepton directly come from the photon fragmentation, they necessarily form a  $^3S_1$  state. There is no helicity suppression mechanism affiliated with this process, consequently no factor of  $m_l$  manifests in (3.1).

For notational abbreviation, we introduce three dimensionless energy variables as  $x_i = 2E_i/M_{\eta_Q} = 2P\cdot p_i/P^2$  ( $i = 1, 2, 3$ ), where  $\sqrt{P^2} = M_{\eta_Q}$ ,  $E_i \equiv p_i^0$  is the energy of each final state particle in the  $\eta_Q$  rest frame. We also adopt the same definition as in the previous section,  $r \equiv 4m_l^2/M_{\eta_Q}^2$ .

It is straightforward to square the amplitude and sum over the polarizations of final-state particles. We then obtain

$$\begin{aligned} \sum|\mathcal{M}|^2 &= \frac{2^9N_c\pi^3e_Q^4\alpha^3}{m_Q^3}\psi_{\eta_Q}^2(0) \\ &\times \frac{[x_1(1-x_1)+x_2(1-x_2)]x_3-2(1-x_1)(1-x_2)+\frac{r}{2}x_3^2}{x_3^2(1-x_3)^2}, \end{aligned} \quad (3.2)$$

which is symmetric under the interchange between  $x_1$  and  $x_2$ . As expected, each of the three terms in the numerator scales as  $x_3^2$  in the infrared limit  $x_3 \rightarrow 0$ .

Energy conservation demands that  $x_1+x_2+x_3=2$ . Any kinematic invariants can be expressed in terms of any pair of these three scaled energy variables. In the following we shall derive the inclusive energy distributions of  $l^-$  and  $\gamma$  separately, hence it is convenient

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<sup>9</sup>By the similar reasoning, it is easy to see that the decay  $\chi_{QJ} \rightarrow l^+l^-\gamma$  does develop an infrared singularity in the soft-photon limit, since the corresponding NRQED operator governing the soft-photon emission is of the electric dipole type, i.e.  $\frac{-iee_Q}{2m_Q}\psi^\dagger[\mathbf{D}\cdot\mathbf{A}^{\text{em}}+\mathbf{A}^{\text{em}}\cdot\mathbf{D}]\psi$ , and there appears no factor of photon momentum to kill the corresponding one in the denominator.

to choose  $x_1$  and  $x_3$  as the two independent variables in expressing the differential three-body phase space. The dimensionless energy variable of  $l^+$ ,  $x_2$ , can be eliminated in the amplitude squared. The energy spectra of  $\gamma$  and  $l^-$  can be obtained by choosing the different order of two-fold integrations over  $x_1$  and  $x_3$ . As a consequence, the decay rate can be expressed in the following two different ways:

$$\Gamma[\eta_Q \rightarrow l^- l^+ \gamma] = \frac{M_{\eta_Q}}{32(2\pi)^3} \int_0^{1-r} dx_3 \int_{x_1^-}^{x_1^+} dx_1 \sum |\mathcal{M}|^2 \quad (3.3a)$$

$$= \frac{M_{\eta_Q}}{32(2\pi)^3} \int_{\sqrt{r}}^1 dx_1 \int_{x_3^-}^{x_3^+} dx_3 \sum |\mathcal{M}|^2. \quad (3.3b)$$

The integration boundaries for the outer-layer integrals have been labeled explicitly; for the inner-layer integral, the upper and lower boundaries can also be readily inferred:

$$x_1^\pm = \frac{2-x_3}{2} \pm \frac{x_3}{2} \sqrt{\frac{1-r-x_3}{1-x_3}}, \quad (3.4a)$$

$$x_3^\pm = \frac{2(1-x_1)}{2-x_1 \mp \sqrt{x_1^2-r}}. \quad (3.4b)$$

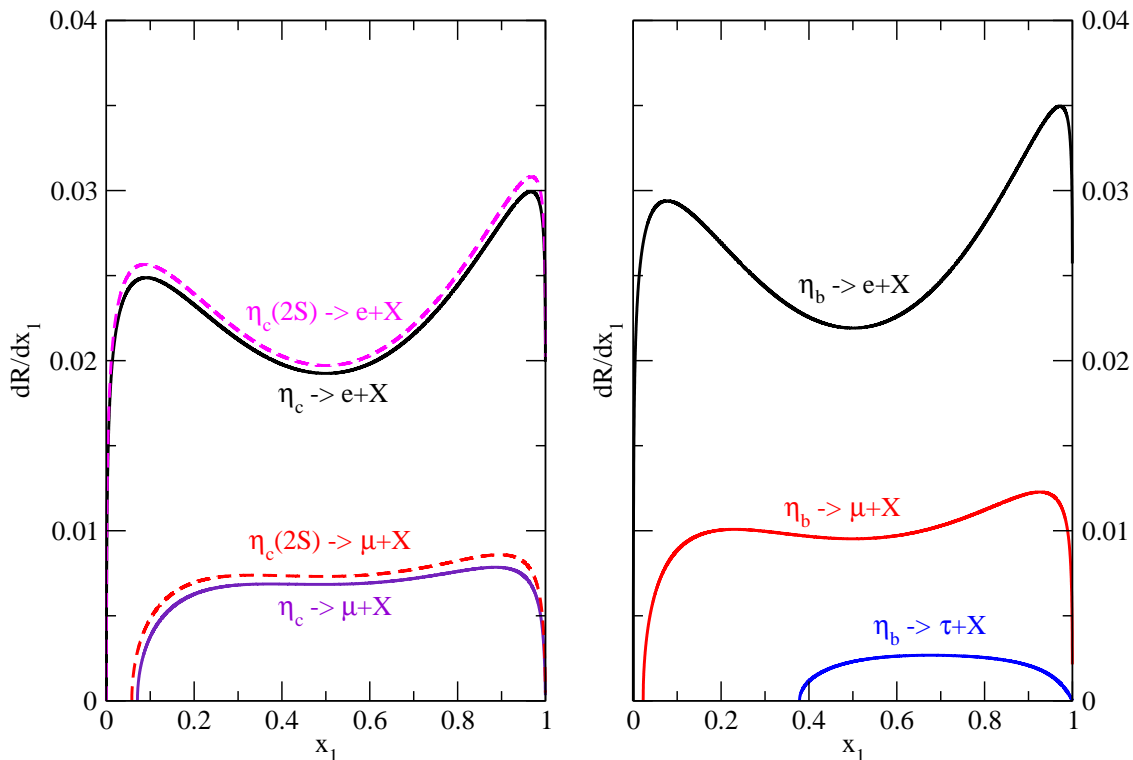
### 3.2 Inclusive photon energy spectrum

It is straightforward to deduce the energy distribution of the photon by integrating over the variable  $x_1$  in (3.3a):

$$\begin{aligned} \frac{dR[\eta_Q \rightarrow \gamma(x_3) + X]}{dx_3} &\equiv \frac{1}{\Gamma_0} \frac{d\Gamma[\eta_Q \rightarrow \gamma(x_3) + X]}{dx_3} \\ &= \frac{\alpha}{3\pi} \frac{x_3(2+r-2x_3)\sqrt{1-r-x_3}}{(1-x_3)^{5/2}}. \end{aligned} \quad (3.5)$$

As in dealing with  $\eta_Q$  decay to a lepton pair, we introduce the scaled energy distribution of  $\gamma$ ,  $dR(x_3)/dx_3$ , the differential decay rate of  $\eta_Q \rightarrow \gamma + X$  normalized with respect to  $\Gamma_0$ , the partial width of  $\eta_Q \rightarrow \gamma\gamma$  given in (2.8). This distribution clearly vanishes at both the lower and upper ends of the scaled photon energy, i.e.  $x_3 = 0$  and  $x_3 = 1-r$ . For  $r \ll 1$ , the spectrum is generally featureless and negligible in most of the region, except a sharp peak rises near the upper end of  $x_3$ , which centers at  $x_3 \approx 1 - \frac{\sqrt{21+1}}{4}r \approx 1 - 1.40r$ , and the peak height  $\approx 0.11\alpha/r$ . This clearly indicates that, the decay rate is dominated by the kinematic configuration where the outgoing lepton pair carries a small invariant mass, owing to the collinear enhancement.

There is also motivation to inspect the lower end of the photon spectrum more closely. It is an experimental fact that a realistic electromagnetic calorimeter can detect photons only down to some minimum limiting energy  $E_{\gamma \text{ cut}}$ . If the photon becomes very soft, it will not be properly registered by the electromagnetic calorimeter. In this respect, those  $\eta_Q \rightarrow l^- l^+ \gamma$  events with a very soft photon will, from the experimental perspective, mimic the respective  $\eta_Q \rightarrow l^- l^+$  event. We learn from (3.5) that, at small  $x_3$ , this differential decay rate becomes enormously suppressed relative to that in the  $x_3 \rightarrow 1$  limit, and scales linearly with  $x_3$ ,  $\sim \frac{\alpha}{3\pi} \sqrt{1-r}(2+r)x_3$ . Imagine we impose a realistic cutoff  $x_{3 \text{ cut}}$  on the photon



**Figure 3.** Normalized energy distributions of leptons in numerous  $\eta_c(\eta'_c)$  and  $\eta_b$  Dalitz decays. The energy spectrum of  $\tau$  in  $\eta'_c$  Dalitz decay, populating a rather narrow region near  $x_1 = 1$ , is very much suppressed with respect to the other ones, so is not displayed in this figure.

energy. Those three-body decay events will be correctly recorded as Dalitz decay events when the fractional photon energy is greater than  $x_{3\text{cut}}$ ; in comparison, those three-body events will be tagged as the lepton pair events when  $x_3 \leq x_{3\text{cut}}$ . The  $R$  value integrating over  $x_3$  from 0 to this cutoff will be approximately  $\frac{\alpha}{3\pi} x_{3\text{cut}}^2$  for small  $r$ . Although this is a tiny fraction, the chance exists that for some Dalitz decays, it might still be much greater than the extremely small decay ratio of  $\eta_Q \rightarrow l^- l^+$  as given in (2.7). If this is the case, what are experimentally recorded as the  $\eta_Q \rightarrow l^- l^+$  events in fact receive the bulk of contributions from the three-body Dalitz decays. As we shall see,  $\eta_c \rightarrow e^- e^+$  constitutes such a very example.

### 3.3 Inclusive lepton energy spectrum and photon-to-lepton fragmentation function

We are also interested in the inclusive energy distribution of the lepton. This can be obtained by integrating over the variable  $x_3$  in (3.3b):

$$\frac{dR[\eta_Q \rightarrow l^-(x_1) + X]}{dx_1} \equiv \frac{1}{\Gamma_0} \frac{d\Gamma[\eta_Q \rightarrow l^-(x_1) + X]}{dx_1}$$

$$\begin{aligned}
 &= \frac{2\alpha}{\pi} \left\{ \frac{x_1^2 + (1-x_1)^2}{2} \ln \left( \frac{x_1 + \sqrt{x_1^2 - r}}{x_1 - \sqrt{x_1^2 - r}} \right) + 2(1-x_1)\sqrt{x_1^2 - r} \right. \\
 &\quad \left. - \frac{1}{2} \ln \left( \frac{2-x_1 + \sqrt{x_1^2 - r}}{2-x_1 - \sqrt{x_1^2 - r}} \right) \right\}. \tag{3.6}
 \end{aligned}$$

As in eq. (3.5), we also define a dimensionless distribution,  $dR(x_1)/dx_1$ , the differential decay rate of  $\eta_Q \rightarrow l^-(x_1) + X$  normalized with respect to  $\Gamma_0$ . It is easy to see that this energy spectrum vanishes at both end points:  $x_1 = \sqrt{r}$  and 1. In sharp contrast to the energy spectrum of  $\gamma$ , the spectrum of  $l^-$  is more evenly populated in the whole region of  $x_1$ . In figure 3, we show the energy distributions of different species of  $l^-$  in various Dalitz decays of  $\eta_c, \eta'_c$  and  $\eta_b$ .<sup>10</sup>

As  $r \ll 1$ , the formalism of fragmentation function can adequately account for the leptonic energy distribution. One can readily identify the fragmentation function  $D_{\gamma \rightarrow l^-}(z)$ , by expanding (3.6) in powers of  $r$ , retaining only those nonvanishing terms in the limit  $r \rightarrow 0$ , and dividing them by 2 to compensate the fact that each photon in  $\eta_Q \rightarrow \gamma\gamma$  can fragment:

$$D_{\gamma \rightarrow l^-}(x_1) = \frac{\alpha}{\pi} \left\{ \frac{x_1^2 + (1-x_1)^2}{2} \ln \left( \frac{4x_1^2}{r} \right) + \frac{1}{2} \ln(1-x_1) + 2x_1(1-x_1) \right\}. \tag{3.7}$$

As expected, the coefficient of the collinear logarithm is nothing but the Altarelli-Parisi splitting kernel  $P_{\gamma \rightarrow l^-}$ . We have checked that, for each Dalitz decay channel  $\eta_Q \rightarrow l^+l^-\gamma$  (except  $l = \tau$ ), this fragmentation function approximates the exact spectra (3.6) quite well, and also gives a satisfactory account of the corresponding integrated decay rates.

### 3.4 The integrated decay rate of $\eta_Q \rightarrow l^+l^-\gamma$

It is also desirable to know the total decay rate of  $\eta_Q \rightarrow l^+l^-\gamma$ , by integrating the amplitude squared in (3.2) over the full three-body phase space. The integrated decay ratio can be readily deduced by starting from either the photon spectrum in (3.5) or the lepton distribution in (3.6). The final answer admits a particularly succinct form:

$$R[\text{integrated}] \equiv \frac{\Gamma[\eta_Q \rightarrow l^+l^-\gamma]}{\Gamma_0} = \frac{2\alpha}{3\pi} \left\{ \ln \left( \frac{1 + \sqrt{1-r}}{1 - \sqrt{1-r}} \right) - \frac{2(4-r)}{3} \sqrt{1-r} \right\}. \tag{3.8}$$

Its limiting behavior in small  $r$  reads

$$R[\text{integrated}] = \frac{2\alpha}{3\pi} \left\{ \ln \left( \frac{1}{r} \right) + 2 \ln 2 - \frac{8}{3} + \mathcal{O}(r) \right\}. \tag{3.9}$$

As expected, the greater the disparity between quark mass and lepton mass is, the larger the integrated  $R$  value becomes due to the increasing collinear enhancement. This asymptotic expression can also be easily reproduced by integrating twice of the fragmentation function  $D_{\gamma \rightarrow l^-}(x_1)$  in (3.7) over the entire range of the energy fraction  $x_1$  of  $l^-$ .

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<sup>10</sup>Note that the quark energy distributions in various  $\eta_Q \rightarrow q\bar{q}g$  processes, have not been correctly displayed in figure 3 of [25], due to some input mistakes.

| Decay modes                           | $r$                   | $f(r)$           | $-g_A^Q \frac{\sqrt{2}G_F M_{\eta_Q}^2}{8e_Q^2 \alpha^2}$ | $R^{\text{EM}}$        | $R^{\text{SM}}$        |
|---------------------------------------|-----------------------|------------------|---|------------------------|------------------------|
| $\eta_c \rightarrow e^+e^-$           | $1.18 \times 10^{-7}$ | $58.67 - 27.24i$ | -0.39   | $1.33 \times 10^{-9}$  | $1.31 \times 10^{-9}$  |
| $\eta_c \rightarrow \mu^+\mu^-$       | $5.03 \times 10^{-3}$ | $5.30 - 10.51i$  | -0.39   | $1.88 \times 10^{-6}$  | $1.82 \times 10^{-6}$  |
| $\eta_c(2S) \rightarrow e^+e^-$       | $7.90 \times 10^{-8}$ | $61.76 - 27.87i$ | -0.58   | $9.79 \times 10^{-10}$ | $9.63 \times 10^{-10}$ |
| $\eta_c(2S) \rightarrow \mu^+\mu^-$   | $3.38 \times 10^{-3}$ | $6.27 - 11.13i$  | -0.58   | $1.49 \times 10^{-6}$  | $1.42 \times 10^{-6}$  |
| $\eta_c(2S) \rightarrow \tau^+\tau^-$ | 0.955                 | $-0.61 - 3.19i$  | -0.58   | $5.78 \times 10^{-6}$  | $6.35 \times 10^{-6}$  |
| $\eta_b \rightarrow e^+e^-$           | $1.19 \times 10^{-8}$ | $77.59 - 30.85i$ | 15.35   | $2.23 \times 10^{-10}$ | $3.07 \times 10^{-10}$ |
| $\eta_b \rightarrow \mu^+\mu^-$       | $5.07 \times 10^{-4}$ | $11.98 - 14.10i$ | 15.35   | $4.68 \times 10^{-7}$  | $1.29 \times 10^{-6}$  |
| $\eta_b \rightarrow \tau^+\tau^-$     | 0.143                 | $0.19 - 5.52i$   | 15.35   | $1.09 \times 10^{-5}$  | $9.74 \times 10^{-5}$  |

**Table 1.** The values of  $r$ , the scaled amplitudes for electromagnetic and weak interactions, and the respective  $R$  ratios for various pseudoscalar quarkonium decays to a lepton pair.  $R^{\text{SM}}$  is the  $R$  ratio that includes both QED and weak contribution, as indicated in (2.7);  $R^{\text{EM}}$  denotes the corresponding  $R$  ratio by retaining only the QED contribution. For some of the decay channels, NRQCD predictions to  $R^{\text{EM}}$  have also been given in [4, 15].

It is also instructive to look at the alternative limit  $r \rightarrow 1$ , where the  $\eta_Q$  is barely heavy enough to disintegrate into a leptons pair at rest plus a zero-energy photon:

$$R[\text{integrated}] = \frac{4\alpha}{15\pi} \{ \beta^5 + \mathcal{O}(\beta^7) \}, \quad (3.10)$$

where  $\beta \equiv \sqrt{1-r}$ . This somewhat unexpectedly severe suppression, will be useful for us to understand the very small decay rate of  $\eta_c(2S) \rightarrow \tau^+\tau^-\gamma$ .

## 4 Phenomenology

In this section we will explore the consequences of the formulas presented in previous sections. In particular, we will analyze numerous leptonic and Dalitz decay channels of pseudoscalar charmonia and bottomonia, to assess their observation potentials in the current and forthcoming high-energy collision facilities.

In table 1, we tabulate numerous predictions for  $\eta_c$ ,  $\eta_c(2S)$  and  $\eta_b$  decays to all possible species of lepton pairs. When evaluating the mass ratio  $r$ , we take the precisely known lepton masses [11]:  $m_e = 0.511$  MeV,  $m_\mu = 105.66$  MeV, and  $m_\tau = 1776.84$  MeV, and use the central values of the less precisely measured pseudoscalar quarkonium masses [11]:  $M_{\eta_c} = 2980.3$  MeV,  $M_{\eta_c(2S)} = 3637.0$  MeV, and  $M_{\eta_b} = 9388.9$  MeV [29]. For the electromagnetic and weak couplings, we take  $\alpha = 1/137$  (for simplicity, we have neglected the running effect of fine structure constant), and Fermi coupling constant  $G_F = 1.166 \times 10^{-5}$  GeV<sup>-2</sup>. We have taken the electric charges of quarks to be  $e_c = 2/3$  and  $e_b = -1/3$ , the weak axial charges of quarks to be  $g_A^c = 1/2$ ,  $g_A^b = -1/2$ , respectively.

Listed in table 1 are various normalized  $R$  values. Since each leptonic decay of  $\eta_Q$  is severely suppressed with respect to  $\eta_Q \rightarrow \gamma\gamma$ ,  $R \ll 1$  certainly is expected. If one wishes to convert these  $R$  ratios to the corresponding branching fractions, one should multiply them by the respective diphoton branching fraction  $\mathcal{B}[\eta_Q \rightarrow \gamma\gamma]$ . For  $\eta_c$ , we may use



the measured result  $\mathcal{B}_{\text{exp}}[\eta_c \rightarrow \gamma\gamma] = 2.4_{-0.9}^{+1.1} \times 10^{-4}$  [11]. For  $\eta_c(2S)$ , only an upper bound has been set experimentally,  $\mathcal{B}_{\text{exp}}[\eta'_c \rightarrow \gamma\gamma] < 5 \times 10^{-4}$ . However, there is good reason to believe  $\mathcal{B}[\eta_c(2S) \rightarrow \gamma\gamma] \approx \mathcal{B}[\eta_c \rightarrow \gamma\gamma]$ . To date the decay  $\eta_b \rightarrow \gamma\gamma$  has not yet been observed experimentally. Theoretically, the branching fraction of  $\eta_Q \rightarrow \gamma\gamma$  can be estimated in NRQCD factorization approach [14]:<sup>11</sup>

$$\begin{aligned} \mathcal{B}[\eta_Q \rightarrow \gamma\gamma] &\approx \frac{\Gamma[\eta_Q \rightarrow \gamma\gamma]}{\Gamma[\eta_Q \rightarrow gg]} & (4.1) \\ &\approx \frac{9e_Q^4\alpha^2}{2\alpha_s^2(2m_Q)} \frac{1 + \left(\frac{\pi^2}{4} - 5\right) C_F \frac{\alpha_s(2m_Q)}{\pi}}{1 + \left[\left(\frac{\pi^2}{4} - 5\right) C_F + \left(\frac{199}{18} - \frac{13\pi^2}{24}\right) C_A - \frac{8}{9}n_f\right] \frac{\alpha_s(2m_Q)}{\pi}}, \end{aligned}$$

where we have approximated the total hadronic width of  $\eta_Q$  by its gluonic width, and have included the NLO QCD corrections for both  $\eta_Q \rightarrow \gamma\gamma$  and  $\eta_Q \rightarrow gg$ .  $C_F = \frac{N_c^2 - 1}{2N_c}$ ,  $C_A = N_c$ , are the Casimirs for the fundamental and adjoint representations of  $SU(3)_c$  group, respectively. Taking the number of active light flavors  $n_f = 4$  for  $\eta_b$ , and  $\alpha_s(2m_b) = 0.18$ , we then get  $\mathcal{B}[\eta_b \rightarrow \gamma\gamma] \approx 4.7 \times 10^{-5}$ .

Owing to the energy conservation,  $\eta_c$  can only decay to  $e^+e^-$  and  $\mu^+\mu^-$ , while  $\eta_c(2S)$  and  $\eta_b$  can access all three generations of leptons. From table 1, one can see that the values of  $r$  span a rather wide range, from the smallest  $10^{-8}$  in  $\eta_b \rightarrow e^+e^-$  to the largest 0.96 in  $\eta'_c \rightarrow \tau^+\tau^-$ . Therefore, quarkonium decays to a lepton pair seem to provide a richer theoretical playground than the analogous decays of light pseudoscalar mesons.

In the following we summarize the main lessons we have learnt from table 1:

1. Among all the studied pseudoscalar quarkonium decays to lepton pair,  $\eta_b \rightarrow \tau^+\tau^-$  seems to have the largest branching ratio,  $\approx 5 \times 10^{-9}$ . However, the number of produced  $\eta_b$  at Super  $B$  experiment may not be copious enough for observing this decay mode. On the other hand, it also looks rather challenging to tag this decay mode in the high-energy hadron collider experiment such as LHC, since the  $\tau$  events are difficult to reconstruct in hadronic collision environment. It is also interesting to note that, the decay  $\eta_b \rightarrow \mu^+\mu^-$ , with a branching ratio as small as  $10^{-10}$ , seems comparable in cleanness with the decay chain  $\eta_b \rightarrow J/\psi J/\psi \rightarrow 4\mu$ , the “golden mode” for hunting  $\eta_b$  at LHC, which has an estimated branching ratio of  $(0.7 - 6.7) \times 10^{-10}$  [31]. It may be worthwhile to look for this dimuon decay mode at LHC, but likely the signal events would be completely swallowed by the copious backgrounds. The rarest decay channels,  $\eta_b \rightarrow e^+e^-$ ,  $\eta_c(\eta'_c) \rightarrow e^+e^-$ , with branching ratios about  $10^{-14} - 10^{-13}$ , seems completely out of the reach of any foreseeable experiments.
2. For light pseudoscalar mesons decays to a lepton pair, one often resorts to the *unitarity bound*, which is obtained from cutting the intermediate photon lines in the amplitude, to estimate the decay rate of  $P \rightarrow l^+l^-$ . In some cases this simplified but model-independent predictions seems not far below the exact results. Inspecting the phase pattern of  $f(r)$  in table 1, it is clear that this approximation can not

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<sup>11</sup>Note that this formula works poorly for  $\eta_c$  decay. For a more satisfactory estimate of  $\mathcal{B}[\eta_c \rightarrow \gamma\gamma]$ , it is important to resum a class of contributions associated with the running of the strong coupling  $\alpha_s$  [30].

| Decay modes                                 | $r$                   | $R^{\text{SM}}[\eta_Q \rightarrow l^+l^-]$ | $R[E_\gamma < 20 \text{ MeV}]$ | $R[\text{integrated}]$ |
|---|-----------------------|--|--------------------------------|------------------------|
| $\eta_c \rightarrow e^+e^-\gamma$           | $1.18 \times 10^{-7}$ | $1.31 \times 10^{-9}$                      | $1.41 \times 10^{-7}$          | 0.0227                 |
| $\eta_c \rightarrow \mu^+\mu^-\gamma$       | $5.03 \times 10^{-3}$ | $1.82 \times 10^{-6}$                      | $1.41 \times 10^{-7}$          | 0.0062                 |
| $\eta_c(2S) \rightarrow e^+e^-\gamma$       | $7.90 \times 10^{-8}$ | $9.63 \times 10^{-10}$                     | $9.44 \times 10^{-8}$          | 0.0233                 |
| $\eta_c(2S) \rightarrow \mu^+\mu^-\gamma$   | $3.38 \times 10^{-3}$ | $1.42 \times 10^{-6}$                      | $9.44 \times 10^{-8}$          | 0.0068                 |
| $\eta_c(2S) \rightarrow \tau^+\tau^-\gamma$ | 0.955                 | $6.35 \times 10^{-6}$                      | $2.73 \times 10^{-8}$          | $2.8 \times 10^{-7}$   |
| $\eta_b \rightarrow e^+e^-\gamma$           | $1.19 \times 10^{-8}$ | $3.07 \times 10^{-10}$                     | $1.41 \times 10^{-8}$          | 0.0263                 |
| $\eta_b \rightarrow \mu^+\mu^-\gamma$       | $5.07 \times 10^{-4}$ | $1.29 \times 10^{-6}$                      | $1.41 \times 10^{-8}$          | 0.0098                 |
| $\eta_b \rightarrow \tau^+\tau^-\gamma$     | 0.143                 | $9.74 \times 10^{-5}$                      | $1.40 \times 10^{-8}$          | 0.0014                 |

**Table 2.** The normalized  $R$  ratios for various pseudoscalar quarkonia Dalitz decays.  $R[E_\gamma < 20 \text{ MeV}]$  represent the corresponding ratio with a 20 MeV cutoff imposed on the photon energy, so these Dalitz events may be experimentally indistinguishable from those  $\eta_Q \rightarrow l^+l^-$  events.  $R[\text{integrated}]$  is the  $R$  value integrated over the whole three-body phase space. For comparison, we also juxtapose the SM predictions to the  $R$  values for pseudoscalar quarkonia decays to a leptonic pair, which are lifted from table 1.

make an accurate account for the majority of pseudoscalar quarkonia leptonic decay processes.

3. The helicity suppression mechanism, which is manifested in the prefactor  $r$  in (2.7), plays a prominent role in dictating the size of each leptonic decay rate, and is much more important than the logarithmically running  $f$  function. This is clearly seen in the smallness of the  $R$  ratio for  $\eta_b \rightarrow e^+e^-$ , even though the respective  $|f|$  is the largest among all the decay channels. In an alternative case,  $\eta_c(2S) \rightarrow \tau^+\tau^-$ , which hardly suffers from helicity suppression because of the rather large  $r$ , the respective branching ratio nevertheless remains small. This may be partly ascribable to the small  $|f|$ , and partly to the rather limited phase space available for this process, recalling the factor  $\beta$  contained in (2.7).
4. The contribution of weak interaction is insignificant in  $\eta_c(\eta'_c)$  decay, but can become important in  $\eta_b$  leptonic decay. For example, including the  $Z^0$  exchange effect will significantly enhance the branching fraction of  $\eta_b \rightarrow \tau^+\tau^-$  predicted by QED alone, almost by one order of magnitude! This can be clearly understood from the fact that, the relative importance of the weak interaction effect grows with  $m_Q$  (see (2.7)). The interference between QED and weak interaction can be either destructive or constructive, depending on the sign of the axial charges of heavy quarks, and also on the sign of the real part of the  $f$  function.

Next we explore the phenomenological consequences of the Dalitz decay  $\eta_Q \rightarrow l^+l^-\gamma$ . In table 2 we tabulate the  $R$  ratios integrated over the full three-body phase space, as well as the corresponding  $R$  ratios by integrating over the photon energy up to 20 MeV. The purpose of including the latter is to assess the likelihood for these Dalitz events to fake the exclusive  $\eta_Q \rightarrow l^+l^-$  event. The main understanding we gained from table 2 are:

1. These three-body Dalitz decay processes in general have branching ratios several orders of magnitude larger than the corresponding  $\eta_Q \rightarrow l^+l^-$  decay. This drastic disparity should be attributed to several factors: less suppression by powers of  $\alpha$ , the absence of helicity suppression, and the collinear enhancement. As a result, the branching ratios of  $\eta_c(\eta'_c) \rightarrow e^+e^-\gamma$  may reach  $4 \times 10^{-6}$ , and those of  $\eta_c(\eta'_c) \rightarrow \mu^+\mu^-\gamma$  may reach  $10^{-6}$ . In the recently launched BESIII experiment, roughly  $10^{10}$   $J/\psi$  events are expected to be accumulated.  $\eta_c$  can be most copiously produced from  $J/\psi$  through the magnetic dipole transition, with  $\mathcal{B}_{\text{exp}}[J/\psi \rightarrow \eta_c\gamma] = 1.3 \pm 0.4\%$  [11]. Therefore about  $10^8$   $\eta_c$  events are expected to be collected, and about  $\mathcal{O}(10^2)$  Dalitz decay events should be produced. Even taking the detection acceptance and efficiency into account, the observation prospect for the aforementioned Dalitz decays at BES-III still looks optimistic.<sup>12</sup> Similarly, if the future Super  $B$  factory can accumulate a huge  $\eta_b$  samples, it may again be feasible to look for the  $\eta_b \rightarrow e^+e^-\gamma, \mu^+\mu^-\gamma$  events. It is obvious that the majority of Dalitz decay events will obey a fragmentation pattern, i.e., a hard photon of  $E_\gamma \approx \frac{M_{\eta_Q}}{2}$  recoiling against a pair of nearly collinear leptons.
2.  $\eta_c(2S) \rightarrow \tau^+\tau^-\gamma$  is the only exceptional Dalitz decay process that has a even smaller decay rate than its respective leptonic decay  $\eta_c(2S) \rightarrow \tau^+\tau^-$ . It is interesting to trace the reason. First by recalling (3.10), we note the rather strong suppression of the integrated decay ratio of the Dalitz decay near the mass threshold,  $\propto \beta^5$ . By contrast, the suppression of the decay  $\eta_c(2S) \rightarrow \tau^+\tau^-$  is only linear in  $\beta$ , stemming from the two-body phase space. It is these two very different threshold scaling behaviors that result in this anomalous pattern.
3. Suppose a realistic electromagnetic calorimeter, say, the one installed in the BES-III detector, can detect those photons only with energy greater than 20 MeV [13]. When such a cutoff is imposed, according to table 2, the number of Dalitz decay events from  $\eta_c(\eta'_c) \rightarrow e^+e^-\gamma$  with photon energy less than this cutoff, turns out to be about 100 times greater than that from  $\eta_c(\eta'_c) \rightarrow e^+e^-$ . Therefore, what are experimentally recorded as the  $\eta_c(\eta'_c) \rightarrow e^+e^-$  events, in fact receive the bulk of contribution from the three-body Dalitz decay events with unregistered soft photons. However, even if incorporating this two orders-of-magnitude enhancement, the corresponding decay probabilities are still too low for such channels to be established at BES-III experiment. A valuable lesson learned from this example is that, in any attempt to interpret the possible  $\eta_Q \rightarrow l^+l^-$  event as the signature for new interactions beyond SM, one must ensure that the contamination from the corresponding Dalitz decays has already been thoroughly understood and carefully incorporated in the analysis.

## 5 Summary and outlook

In this work, we have performed a comprehensive analysis of pseudoscalar quarkonium decays to a lepton pair without and with bremsstrahlung. These rare electromagnetic decay

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<sup>12</sup>However, the decay  $\eta_c \rightarrow e^+e^-\gamma$  at BES-III experiment may be subject to substantial contamination from the background Bhabha events. In comparison, the decay  $\eta_c \rightarrow \mu^+\mu^-\gamma$  might be easier to establish.

processes offer a clean platform to test our understanding of quarkonium dynamics. We corroborate the previous conclusion that the exclusive decays to lepton pair are extremely suppressed in Standard Model. By contrast, the pseudoscalar quarkonium Dalitz decays in general have a much larger decay rate, because of several joint factors: less suppression by power of  $\alpha$ , absence of helicity suppression, and collinear enhancement. It is found that the Dalitz decays  $\eta_c \rightarrow e^+e^-\gamma$  and  $\eta_c \rightarrow \mu^+\mu^-\gamma$ , with branching fractions of order  $10^{-6}$ , may have the bright prospect to be established in the BES-III experiment.

It is stressed that a realistic electromagnetic calorimeter can detect photons only down to a minimum energy. Thus from the experimental perspective, those Dalitz decay events with photon energy less than this minimum energy, will be tagged as the exclusive lepton pair events. Taking this fact into consideration, it is found that the *measured* decay rate of  $\eta_c(\eta_c') \rightarrow e^+e^-$  would be about two orders of magnitude greater than that *literally predicted* from (2.7). Nevertheless, such amplification is still not dramatic enough to warrant their observation at BES-III experiment. In general, the observation prospect for the pseudoscalar quarkonium decays to a lepton pair seems extremely pessimistic within the frame of the Standard Model. In this respect, future unambiguous sighting of any of this type of decays may be viewed as the strong evidence for the existence of new physics.

Our analysis is based on a leading order calculation both in strong coupling and in quark velocity. To improve the reliability of our predictions, it is worthwhile to implement the QCD perturbative and relativistic corrections to the pseudoscalar quarkonium decay processes considered in this work, in particular to the quarkonium Dalitz decay processes.

An interesting extension of this work is to study  $^3P_J$  ( $J = 0, 1, 2$ ) quarkonium states decays to a lepton pair with and without bremsstrahlung. The processes  $\chi_{c1,2} \rightarrow e^+e^-$  and  $e^+e^- \rightarrow \chi_{c1,2}$  have been studied long ago within the color-singlet model [32],<sup>13</sup> where infrared divergences are reported and subsequently cured by imposing a phenomenological cutoff of “binding energy”. To our knowledge, the process  $\chi_{cJ} \rightarrow l^+l^-\gamma$  has not been considered before, which is also plagued with infrared divergences. It is theoretically interesting to investigate these two processes in the context of NRQCD (NRQED), which provide a systematic way to tame these infrared divergences by incorporating the effect of higher Fock state ( $|c\bar{c}(^3S_1)\gamma\rangle$  in our case, with the dynamical photon understood to be *ultrasoft*). We note that the latter process is quite analogous to the  $\chi_{bJ}$  inclusive decays to charmed hadrons, which have recently been analyzed in the NRQCD factorization approach [33]. Thus one may simply use their results with some slight modification.

Eliminating the infrared divergences associated with the former process turns out to be more subtle, and, more interesting, but likely to be achievable provided that one appeals to the even lower-energy effective theory of NRQCD, the so-called potential NRQCD (pNRQCD) [34], by retaining the ultrasoft gluons (photons) as the manifest degree of freedom<sup>14</sup> (for a pNRQCD-based study of the color-octet effect in exclusive reaction involving

<sup>13</sup>For the calculations of  $\chi_{c1,2} \rightarrow e^+e^-$ , one is allowed to put  $m_e$  to 0 since these processes are not subject to the helicity suppression, and retaining a nonzero  $m_e$  only yields a small correction.

<sup>14</sup>The symptom encountered here seems to be of the similar origin as what was found in the exclusive  $B$ -meson decays to  $P$ -wave charmonium, e.g.  $B \rightarrow \chi_{cJ}K$  [35]. In that case, it has been recently shown that under certain assumptions, the factorization can be recovered provided that the color-octet contribution is

quarkonium, such as  $J/\psi \rightarrow \eta_c \gamma$ , see [37]). We hope future studies of these two types of processes at BES-III experiment, especially the resonant production process  $e^+e^- \rightarrow \chi_{c1,2}$ , may lend some important guidance.

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